The Effect of Shadowing on Cell Residence Time in Mobile LEO Satellite Cellular Systems

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Abstract. In previous literature, several cell residence time models for mobile LEO satellite cellular system (MLSCS) were presented. However, the effects of signal-impairing factors such as shadowing and fading on the presented models were neglected. In this paper, the effects of signal-impairing factors on cell residence time model are investigated. In particular, the probability density function of the cell radius and the probability density function of the maximum cell residence time for MLSCS in the presence of shadowing are derived. Then, we study the performance of MLSCS in the presence of signal-impairing factors. Specifically, we evaluate the probability of premature call termination due to handoff failure and the probability of call dropping due to handoff failure or new call blocking in the presence of signal-impairing factors. The analytical results are validated by computer simulation.

Keywords: Cell Residence Time, Handoff, Shadowing, Premature Call Termination Probability, Received Signal Power.

1. Introduction

In MLSCS the cell residence time is defined as the time a customer spends in a cell, and since the speed of the LEO satellite is very high compared to the speed of the mobile it is widely assumed that the mobility in MLSCS is due solely to the satellite velocity. Based on the above assumption, it is obvious that the cell residence time in MLSCS is finite since the satellite travels in a fixed orbit. Therefore, the statistical distributions usually used to describe the cell residence time in mobile LEO satellite cellular networks support a finite range [0, *maximum cell* *residence time*]. Therefore, the maximum cell residence time (τ) in a MLSCS where ideal channels are assumed (no signal impairing) is a function of the speed of the satellite V_S and the radius of the cell R_C and is defined as [1-2]

$$\tau = \frac{2R_C}{V_S} \tag{1}$$

In previous literature, several MLSCS cell residence time models were presented [1-3]. However, the effect of signal-impairing factors on the presented models (hence, on the performance measures) were neglected. In this paper, the effects of signal-impairing factors on cell residence time model are investigated, and then, we study the performance of MLSCS in the presence of signal-impairing factors. In particular, we evaluate the probability of premature call termination (denoted by P_{ct}) and the probability of call dropping (denoted by P_{cd}) due to handoff failure and call blocking. P_{ct} may be defined as the probability that a call will be dropped during its lifetime due to handoff failure, while P_{cd} is the probability that a call will be dropped either due to new call blocking because of lack of resources or due to premature call termination due to handoff failure [2-4].

2. Cell Radius Calculation

In MLSCS the handoff decision is made based on the received signal power from the current cell and the distance from the center of the cell, where a handoff from the current cell to next cell is requested whenever the received signal power from the current cell falls below a prespecified threshold P_{Th} or when the distance from the center of the cell exceeds R_C [5]. Therefore, a mobile user at distance *R* from the center of *i-th* cell may be considered inside that cell as long as the following conditions are satisfied:

$$P_i(R) \geq P_{Th} \quad \text{and} \quad R \leq R_C \tag{2}$$

where $P_i(R)$ is the received signal power from cell *i* at distance *R* from the center of cell. Hence, the radius of the cell may be defined as the distance R_C that satisfies the following condition

$$P_i(R_C) = P_{Th} \tag{3}$$

In the case of an ideal channel (no signal-impairing factors), assuming circular cell shape, all mobile users located at distance *R* from the center of the cell receive the same signal power level. Hence, it follows that [5]

$$P_i(R) = \frac{P_t G_t G_r \lambda_w^2}{(4\pi)^2 \left(h^2 + R^2\right)} \tag{4}$$

where P_t is the transmitted signal power, *h* is the altitude of the satellite, λ_w is the transmission wavelength, G_t is the transmitter gain, and G_r is the receiver gain. Solving (3) and (4) for R_c we have

$$R_c = \sqrt{C - h^2} \tag{5}$$

where $C = \frac{P_t G_t G_r \lambda_w^2}{(4\pi)^2 P_{Th}}$

3. The Effect of Shadowing on Cell Residence Time

From Section 2, one may observe that for all points R $\left(R \le R_c\right)$ in cell *i*, the received signal should exceed the power threshold, *i.e.*, $P_i\left(R\right) \ge P_{Th}$ for $\forall\ R \le R_c$ as shown in Fig. 1. In the presence of shadowing the received signal power at distance *R* from the center of cell *i* may be found as [5]

$$P_i\left(R\right) = \frac{P_t G_t G_r \lambda_w^2}{(4\pi)^2 \left(h^2 + R^2\right)} \zeta_i \quad \text{where} \quad P_i\left(R\right) \ge P_{Th} \ \text{ for } \forall\ R \le R_c \tag{6}$$

where ζ_i is a log-normal random variable with parameters σ and μ that represents the shadowing effect with probability density function (pdf), mean, and variance defined, respectively, as

$$f\left(\zeta_i\right) = \frac{1}{\sigma\sqrt{2\pi}\zeta_i} e^{\frac{-\left(\ln(\zeta_i) - \mu\right)^2}{2\sigma^2}} \tag{7}$$

$$E\left[\zeta_i\right] = e^{\left(\mu + \frac{\sigma^2}{2}\right)} \tag{8}$$

and

$$\sigma_{\zeta_i}^2 = e^{\left(\sigma^2 + 2\mu\right)}\left(e^{\sigma^2} - 1\right). \tag{9}$$

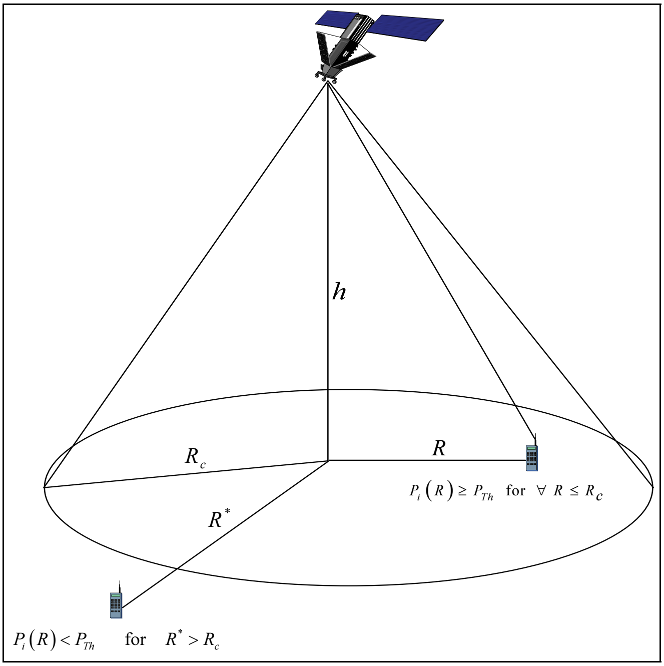

Fig. 1. Cell geometry and handoff conditions in MLSCS.

At threshold, (Similar to (5)) the actual cell radius in the presence of shadowing R_s may be found from (6) as

$$R_S = \sqrt{C\zeta_i - h^2} \qquad 0 \le R_S \le R_C \qquad (10)$$

then

$$R_s = \begin{cases} 0 & \zeta_i \le \dfrac{h^2}{C} \\ \sqrt{C\zeta_i - h^2} & \dfrac{h^2}{C} < \zeta_i < \dfrac{h^2 + R_c^2}{C} \\ R_c & \zeta_i \ge \dfrac{h^2 + R_c^2}{C}, \end{cases} \qquad (11)$$

where $C = \dfrac{P_t G_t G_r \lambda_w^2}{(4\pi)^2 P_{Th}}$. Since ζ_i is a log-normal random variable, the pdf of R_s may be found using random variable transformation as [6]

$$f_{R_s}(r)=\begin{cases}\Pr\left(\zeta_i \le \frac{h^2}{C}\right)\delta(r) & r=0\\ f_{\zeta_i}\left(\frac{h^2+r^2}{C}\right)\left|\frac{d\zeta_i}{dR_s}\right| & 0<r<R_c\\ \Pr\left(\zeta_i \ge \frac{h^2+r^2}{C}\right)\delta(r-R_c) & r=R_c,\end{cases} \tag{12}$$

Where $\delta(r)$ is the Dirac delta function. It follows from (12) that the pdf of the cell radius in the presence of shadowing is given by

$$f_{R_s}(r)=\begin{cases}\frac{1}{2}erfc\left(\frac{\mu-ln\left(\frac{h^2}{C}\right)}{2\sqrt{\sigma}}\right)\delta(r) & r=0\\ \frac{2r}{\sigma(r^2+h^2)\sqrt{2\pi}}\exp\left(\frac{-\left(ln\left(\frac{r^2+h^2}{C}\right)-\mu\right)^2}{2\sigma^2}\right) & 0\le r\le R_c\\ 1-\frac{1}{2}erfc\left(\frac{\mu-ln\left(\frac{h^2+R_c^2}{C}\right)}{2\sqrt{\sigma}}\right)\delta(r-R_c) & r=R_c,\end{cases} \tag{13}$$

where $erfc[x]=\frac{2}{\sqrt{\pi}}\int_x^{\infty}e^{-t^2}dt$ is the complementary error function. The pdf of the maximum cell residence time in the presence of shadowing τ_s may be found from (13) using random variable transformation as [6]

$$f_{\tau_s}(t)=f_{R_s}(r)\left|\frac{dR_s}{d\tau_s}\right|, \tag{14}$$

where from (1), we have

$$R_s=\frac{\tau_s V_s}{2} \quad \text{and} \quad \frac{dR_s}{d\tau_s}=\frac{V_s}{2}. \tag{15}$$

Then

$$f_{\tau_s}(t)=\begin{cases}\frac{1}{2}Erfc\left(\frac{\mu-ln\left(\frac{h^2}{C}\right)}{2\sqrt{\sigma}}\right)\frac{V_s}{2}\delta(t) & t=0\\ \frac{V_s^2 t}{2\sigma\left(\frac{(V_s t)^2}{4}+h^2\right)\sqrt{2\pi}}\exp\left\{-\left(ln\left[\frac{\frac{(V_s t)^2}{4}+h^2}{C}\right]-\mu\right)^2\left(\frac{1}{2\sigma^2}\right)\right\} & 0<t<\frac{2R_c}{V_s}\\ 1-\frac{1}{2}Erfc\left(\frac{\mu-ln\left(\frac{h^2+R_c^2}{C}\right)}{2\sqrt{\sigma}}\right)\frac{V_s}{2}\delta\left(t-\frac{2R_c}{V_s}\right) & t=\frac{2R_c}{V_s}.\end{cases} \tag{16}$$

Hence, the *k-th* moment of the residence time in the *i-th* cell (i=0,1,…) may be found as

$$E\left[T_i^{\,k}\right]=\int_0^{\frac{2R_c}{V_s}}\int_0^{\tau_s} t^k f_{T_i}(t) f_{\tau_s}(\tau)\,dt\,d\tau \tag{17}$$

where T_i is a random variable denoting the residence time in the *i-th* cell. Based on the effect of shadowing on cell residence time, it is possible to determine the effect of shadowing on several performance measures such as premature call termination probability and call dropping probability in the presence of shadowing, which will be discussed in the next section.

4. Illustration and Results

To illustrate the effect of shadowing on the cell residence time, the cell residence time model presented in Ref. [2] is used. The model presented in Ref. [2] consists of two components, the first component represents the residence time in the origination cell T_0, and the second represents the residence time in subsequent cells T_i (*i=1,2,3,…*), where [2]

$$f_{T_0}(t)=\begin{cases}\frac{4}{\pi\tau_s}\sqrt{1-\left(\frac{t}{\tau_s}\right)^2} & 0\le t\le\tau_s\\ 0 & elsewhere\end{cases} \tag{18}$$

and

$$f_{T_i}(t)=\begin{cases}\dfrac{t}{\tau_s^2\sqrt{1-(t/\tau_s)^2}} & 0 \le t < (1-\dfrac{\varepsilon}{2})\tau_s \\ \sin(\cos^{-1}(1-\varepsilon/2)) & t = (1-\dfrac{\varepsilon}{2})\tau_s \\ 0 & elsewhere,\end{cases} \tag{19}$$

where ε is the overlap parameter. In Fig. 2, the effect of shadowing on the means of cell residence time in the origination cell and in subsequent cells is shown, where from (17), (18), and (19) we have

$$E\left[T_0^k\right]=\int_0^{\frac{2R_c}{V_s}}\int_0^{\tau_s} t^k f_{T_0}(t) f_{\tau_s}(\tau)\,dt\,d\tau \tag{20}$$

and

$$E\left[T_i^k\right]=\int_0^{\frac{2R_c}{V_s}}\int_0^{\tau_s\left(1-\frac{\varepsilon}{2}\right)} t^k f_{T_i}(t) f_{\tau_s}(\tau)\,dt\,d\tau\,. \tag{21}$$

The figure shows that as the shadowing increases the mean of the cell residence time decreases, this is because increasing the shadowing decreases the received signal power from the current cell, hence, a handoff will be requested at distance $R_S \le R_C$ (hence the maximum cell residence time $\tau_S \le \tau$). The effect of shadowing on premature call termination probability (P_{ct}) is presented in Fig. 3. From Ref. [7] we have

$$P_{ct} \cong \sum_{j=0}^{N} p_{hf}\left(1-p_{hf}\right)^j\left\{1-F_H\left(\mu_{G_j}\right)-\frac{\sigma_{G_j}^2}{2q^2}\left[F_H\left(\mu_{G_j}+q\right)-2F_H\left(\mu_{G_j}\right)+F_H\left(\mu_{G_j}-q\right)\right]\right\} \tag{22}$$

where P_{fh} is the handoff failure probability, that is, the probability that a handoff request will be denied [2-3], [8-11], $F_H(t)$ is the cumulative distribution function (cdf) of the random variable (H) which represents the call holding time, G_j, μ_{G_i}, and $\sigma_{G_j}^2$ are defined, respectively, as $G_j = T_0 + \sum_{i=1}^{j} T_i$, $\mu_{G_j} = \mu_{T_0} + \sum_{i=1}^{j}\mu_{T_i}$ and $\sigma_{G_j}^2 = \sigma_{T_0}^2 + \sum_{i=1}^{j}\sigma_{T_i}^2$ where μ_x and

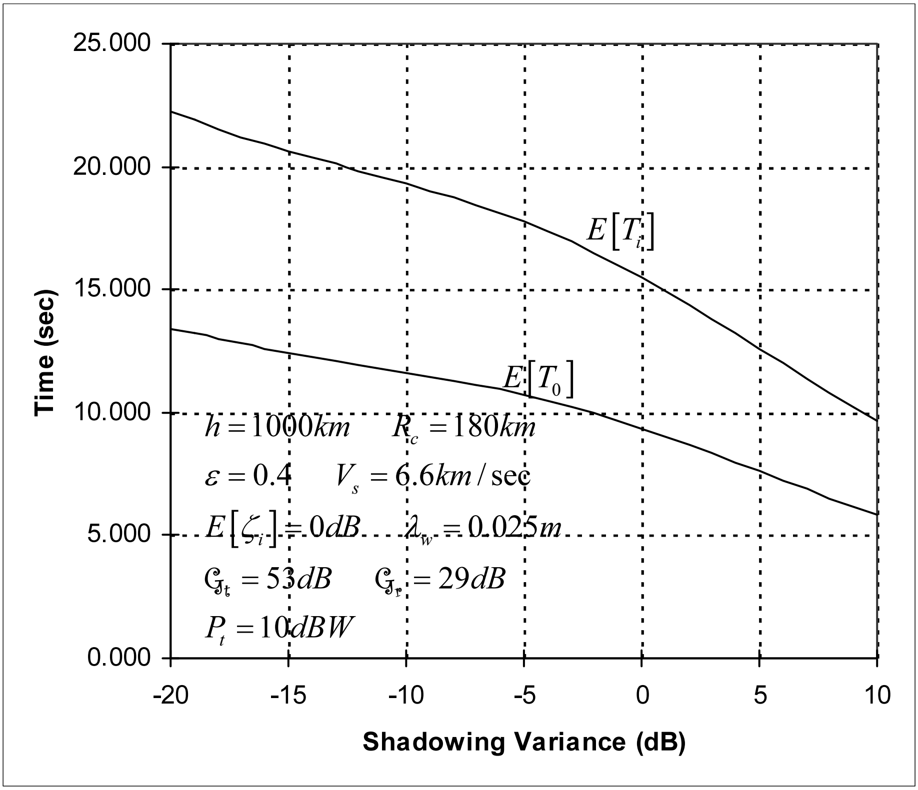

Fig. 2. Mean of the cell residence time vs the variance of the shadowing.

σ_x^2 are the mean and the variance of the random variable x respectively, and the difference parameter q may be found as shown in Ref. [7]. Several statistical distributions (such as the Erlang, gamma, and lognormal distributions) have been used in the literature to model the call holding time in MLSCS and PCS networks. We note that many of these distributions are special or limited cases of the more versatile generalized gamma distribution. Therefore, we assume further that the call holding time follows a three-parameter generalized gamma distribution with pdf given by [12]

$$f_H(t) = \frac{ct^{\alpha c-1} e^{-\left(\frac{t}{\beta}\right)^c}}{\beta^{\alpha c}\Gamma(\alpha)} \tag{23}$$

where α is the shape parameter, β is the scale parameter, and c is the power parameter. The corresponding cdf is given by

$$F_H(t) = \frac{\gamma\left(\alpha, (t/\beta)^c\right)}{\Gamma(\alpha)} \tag{24}$$

where $\gamma(x,y) = \int_0^y t^{x-1} e^{-t} dt$ is the incomplete gamma function and $\Gamma(x)$ is the gamma function [12]. In Fig. 3, where $\alpha = 3$, $\beta = 60$, and $c = 1$, one may observe that the premature call termination probability increases as the shadowing increases, this is due to the fact that as the shadowing increases the maximum cell residence time reduces as shown in Fig. 2, hence, the call will pass through a larger number of cells (larger number of handoff requests) during its lifetime. Therefore, the probability of premature call termination during its lifetime due to handoff failure increases as the shadowing increases. The effect of shadowing on call dropping probability is shown in Fig. 4, where

$$P_{cd} = p_n + (1 - p_n) P_{ct}, \tag{25}$$

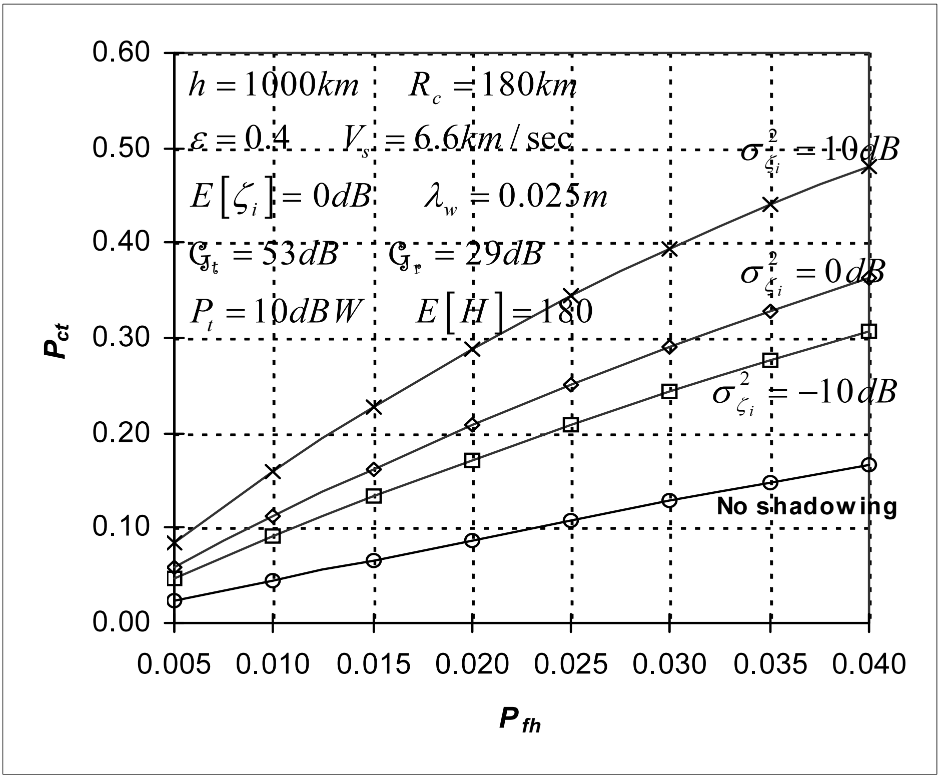

Fig. 3. Premature call termination probability vs handoff failure probability, for different values of the shadowing factors variance.

and p_n denotes the new call blocking probability. As expected, the call dropping probability increases as the shadowing increases. This is due to the fact that premature call termination probability increases as the shadowing increases, as shown in Fig. 3, which, in turn, will increase the probability of call dropping.

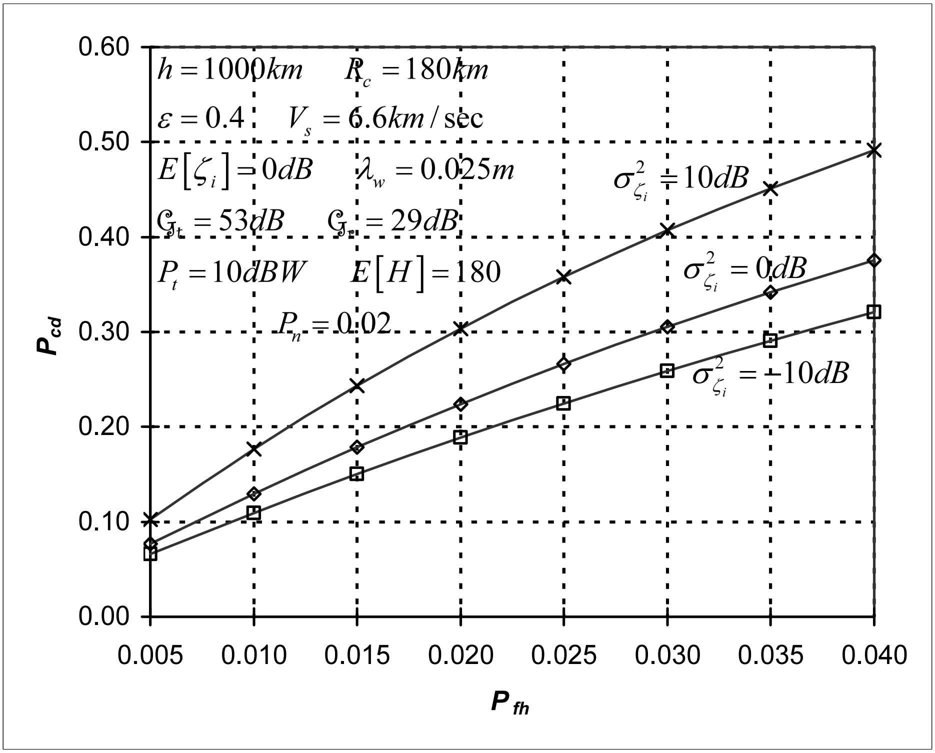

Fig. 4. Call dropping probability vs handoff failure probability, for different values of the shadowing factors variance.

5. Conclusion

In this paper, we investigated the effect of shadowing on cell residence time and on the performance of MLSCS. In particular, the pdf of the cell residence time, and an expression to determine the *k-th* moment of cell residence time in MLSCS in the presence of shadowing were derived. We showed that shadowing has a significant impact on the performance of such a system. In addition, we showed that the mean value of the cell residence time distribution has a significant influence on the performance of MLSCS.

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تأثير التظليل (الحجب) على زمن البقاء في الخلايا في الأنظمة المتنقلة المرتبطة بالأقمار الصناعية منخفضة المدار

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المستخلص: في الدراسات السابقة تم تقديم العديد من نماذج زمن البقاء في الخلايا في الأنظمة المتنقلة المرتبطة بالأقمار الصناعية منخفضة المدار. إلا أن النماذج المقدمة لم تشمل تأثير العوامل المؤثرة في الإشارات كالتظليل (الحجب) على زمن البقاء في الخلايا. تم في هذا البحث دراسة تأثير العوامل المؤثرة في الإشارات على زمن البقاء في الخلايا، وعلى وجه التحديد، تم اشتقاق دالة الاحتمالات الخاصة بنصف قطر الخلية، بالإضافة إلى اشتقاق دالة الاحتمالات الخاصة بالحد الأقصى لزمن البقاء في الخلايا، مع افتراض وجود تأثير التظليل على الإشارة المستخدمة في قناة الاتصال. كما يقدم هذا البحث دراسة لأداء الأنظمة المتنقلة المرتبطة بالأقمار الصناعية منخفضة المدار، مع افتراض وجود تأثير التظليل على الإشارة المستخدمة في قناة الاتصال. حيث يتم تحديدًا دراسة احتمالية إنهاء المكالمة قسريا قبل اكتمالها نتيجة خطأ في التسليم عند نقل المكالمة من خلية إلى أخري، بالإضافة إلى دراسة احتمالية إسقاط المكالمة الناتج عن إنهاء المكالمة قسريا أو عن عدم قبولها كمكالمة جديدة نظرا لمحدودية مصادر النظام. وتم التحقق من نتائج نموذج التحليل بمقارنتها بالنتائج التي تم الحصول عليها من برامج المحاكاة.